(To aid understanding of this article by those of a nervous mathematical disposition, I have added in some explanatory notes. Any lack of clarity in the glossary is therefore entirely my own. ~ *MH*)

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Random Thoughts on *phi*

By Sig Lonegren

Over the past twenty-five years, I have found that my interest in phi, (Ø), 1.648:1 has waned and ebbed. Recently I have been drawn back in to taking another look at this fascinating number, and at the other sacred geometrical numbers: π (3.1416…), √2 (1.414…), √3 (1.732…), √5 (2.236…), and Euler's natural logarithm *e* (2.72)[i]. I have been finding interesting ways that most of them seemthem seem to interrelate numerically.

But first, back to the basics, and some definitions:

An **irrational number** is a number that cannot be expressed as a fraction or ratio of any two whole numbers. Irrational numbers have decimal expansions that neither terminate nor become periodic. (Thus, for example, 10/3 = 3.333333etc is not irrational.) Every transcendental number is irrational.

I have never really understood what the definition of a transcendental number is as I have dyslexia, failed most math courses in school, and have no idea of what an "integer polynomial" or "an algebraic number of any degree" is, but here goes:

A **transcendental number**[ii] is a number that is not the root of *any* integer polynomial[iii], meaning that it is not an algebraic number[iv] of any degree[v]. This definition guarantees that every transcendental number must also be irrational, since a rational number is, by definition, an algebraic number of degree one.

Leonhard Euler was born at Basle, Switzerland on April 15, 1707, and died at St. Petersburg, Russia, on September 7, 1783. His symbol **e (epsilon)** is used to denote the base of the Napierian or natural logarithms, namely, the incommensurable[vi] number 2.71828... (2.72), and the symbol π (pi) used to denote the incommensurable number 3.14159....(3.1416). Newton had been the first to employ the literal exponential notation[vii], and Euler, using the form, had taken *a* as the base of any system of logarithms. It is probable that the choice of *e* for a particular base was determined by its being the vowel consecutive to *a.* A Napierian logarithm is a logarithm to the base *e* or 2.72

2.72 is the number of feet in Alexander Thom's Megalithic Yard. 272 is the number of days in a nine month human gestation period (30 x 9 = 270; 365/12 x 9 = 273.75; the average of these two numbers 270 +273.75/ = 543.75. 543.75/2 = 271.875, which rounds up to 272. Some have said that there are 272 stones in the Chartres Labyrinth, but this has been disputed – it's a bit like the different counts of numbers of stones in a number of different stone circles.

The Fibonacci Series

In this series, each number is the sum of the two numbers preceding it.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, etc.

One big question for me is what number does this series actually start with? First a word about zero (0). The Italian mathematician Leonardo Fibonacci (1170 – 1250) was one of the main people who brought the concept of zero to Europe.

In *Liber Abaci* he described the nine Indian symbols together with the sign 0 for Europeans in around 1200 but it was not widely used for a long time after that. It is significant that Fibonacci was not bold enough to treat 0 in the same way as the other numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 since he speaks of the "sign" zero while the other symbols he speaks of as numbers. (We need to get down on our knees each night and thank Fibonacci because, in effect, he got rid of Roman n~~N~~umerals! But that is a different story.)

But Brother Fibonacci apparently did start his famous series with zero, and most references to his series do begin with that cipher. But I must admit to having great difficulty with this in that in the purpose of sacred geometry is to take us back to the One. So I have chosen to start the series with one (1) instead of zero (0).

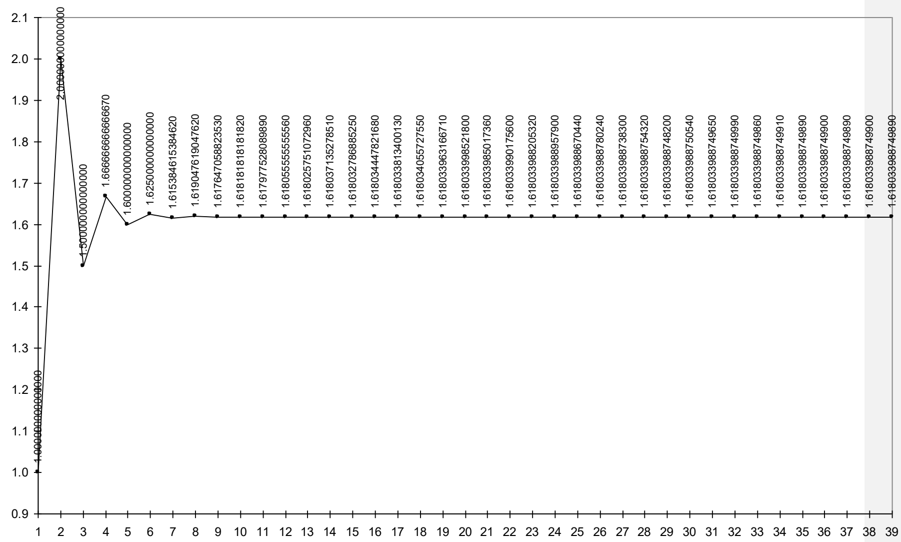
For any value larger than 3 in the sequence, if you divide any two consecutive numbers in the Fibonacci series, the answer is always an approximation of *phi,* or the Golden Ratio,

Fibonacci Numerator	Fibonacci Denominator	Number in Series	Result of Division
1	1	1	1.000000000000000
2	1	2	2.000000000000000
3	2	3	1.500000000000000
5	3	4	1.666666666666670
8	5	5	1.600000000000000
13	8	6	1.625000000000000
21	13	7	1.615384615384620
34	21	8	1.619047619047620
55	34	9	1.617647058823530
89	55	10	1.618181818181820
144	89	11	1.617977528089890
233	144	12	1.618055555555560
377	233	13	1.618025751072960
610	377	14	1.618037135278510
987	610	15	1.618032786885250
1597	987	16	1.618034447821680
2584	1597	17	1.618033813400130
4181	2584	18	1.618034055727550
6765	4181	19	1.618033963166710
10946	6765	20	1.618033998521800
17711	10946	21	1.618033985017360
28657	17711	22	1.618033990175600
46368	28657	23	1.618033988205320
75025	46368	24	1.618033988957900
121393	75025	25	1.618033988670440
196418	121393	26	1.618033988780240
317811	196418	27	1.618033988738300
514229	317811	28	1.618033988754320
832040	514229	29	1.618033988748200
1346269	832040	30	1.618033988750540
2178309	1346269	31	1.618033988749650
3524578	2178309	32	1.618033988749990
5702887	3524578	33	1.618033988749860
9227465	5702887	34	1.618033988749910
14930352	9227465	35	1.618033988749890
24157817	14930352	36	1.618033988749900
39088169	24157817	37	1.618033988749890
63245986	39088169	38	1.618033988749900
102334155	63245986	39	1.618033988749890

Notice ~~how the higher you go~~as you go higher up in the series how the differences between the results of the division become less and less as the series gets closer and closer to the irrational number that it can ~~not~~ never reach. For example, in the last nine numbers listed in the series above, the results are the identical out to nine decimal places!

And yet, in the charts below, when we graph these numbers, notice how, at the beginning, they vary widely from *phi,* or 1.618.

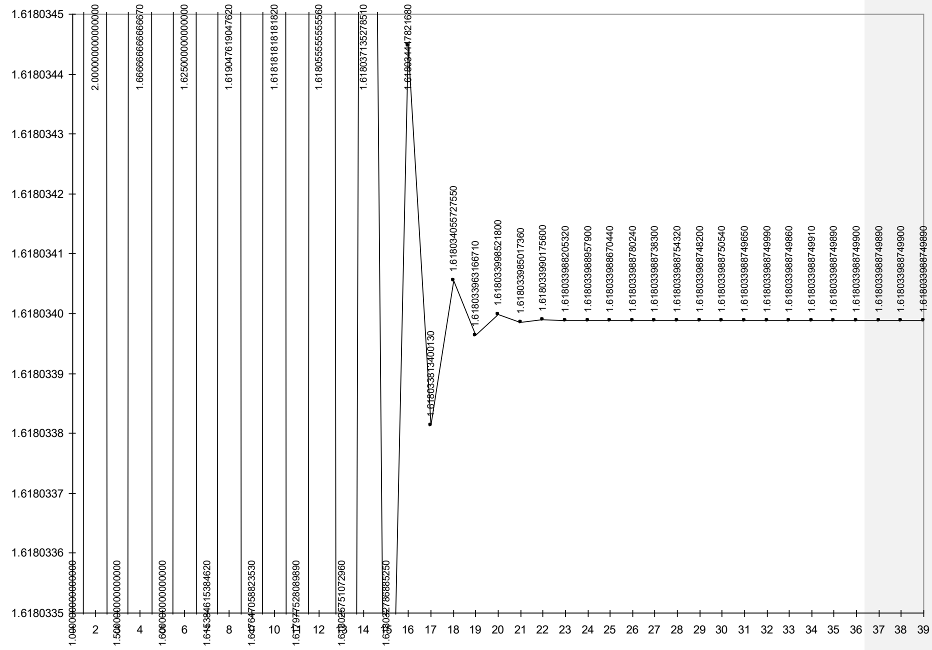
The Fibonacci Series



The Division of the Fibonacci Numbers Shown Graphically
1/1, 2/1, 3/2, 5/3, 8/5, 13/8 etc. up to 39 places

This characteristic of being asymptotic reminds me of a guitar string vibrating around a specific pitch – in this case, *phi.*

We now zoom in on a narrow area around the asymptotic value that we have set at 1.61803398875.



Compare The first chart with the second. This appears to be a fractal phenomenon. The first chart is slightly distorted because of the initial numbers in the series – 1, 2 & 3 cause such significant deviation from the value of *phi.* But they do look remarkably the same!

Some interesting mathematical aspects of *phi:*

Ø = 1.618
Ø^2 = 1.681 + 1 (*phi* is the only number whose square is exactly one more than the number.)
Ø = 1/Ø + 1

Some interesting mathematical connections between *phi* and other sacred geometrical numbers:

√ Ø = 1.272
e = √ Ø – 1 x 1/10

√ 5 = 1.618 + .618, or 2.2360

987 is the 15th number in the Fibonacci Series.
√987 = 31.416
π = 3.14159

While I am aware that one can begin geometrically with the circle and then the *vesica piscis*, and one can then create all of the other geometric shapes that yield the other sacred geometrical ratios, the square (√2), the double square (√ 5) and *phi* in a pentacle (pentagram) or five pointed star, I have not yet discovered a way to connect the √3 (as found in the *vesica*) to the other ratios as shown above.

This paper is a list of things I have been noticing about *phi* and other sacred geometrical ratios. I trust it has triggered some new ways of looking at these numbers for you.

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[i] A **logarithm** is a number which shows how many times any number, called the base, has to be multiplied by itself (in other words, be raised) to produce another number, e.g. the logarithm of 125 to the base 5 is 3 as in $5^3=125$ $(5 \times 5 \times 5 = 125)$
The **natural logarithm**, sometimes known as the **Napierian logarithm**, is the logarithm to the base ***e*** (2.72)

[ii] A **transcendental** number is an irrational number that is not algebraic (see [iv] below)

[iii] An **integer** is a natural number (any finite cardinal or ordinal number), or its negative, or zero, i.e, -3,-2,-1,0,1,2,3…
A **polynomial** (sometimes called **multinomial**) is an algebraic expression consisting of one or more summed terms, each term consisting of a constant multiplier and one or more variables raised to an integral power, e.g, $x^2 - 5x + 6$, or $2p\ 3q + y$

[iv] An **algebraic number** is a real number (a number that has a non-repeating decimal expansion, i.e., it cannot be represented by any ratio of integers) for which there exists a polynomial equation with natural numbers or their negatives as coefficients (the quantity placed before and multiplying the variable in an algebraic expression, e.g.,8 in $8x^2$) such that the given real number is a solution.
Examples of algebraic numbers are all natural numbers, all rational numbers, and some, but not all, irrational numbers

[v] In other words, although some irrational numbers can be easily defined by an equation such as √5, transcendental numbers cannot.
[vi] ~~An **incommensurable** number is a number having an irrational ratio; in other words, that is in a ratio that cannot be expressed by means or integers~~Incommensurable is another word for irrational, literally meaning 'cannot be measured'.

[vii] The **exponent** is a symbol written above and to the right of a mathematical expression to indicate the operation of raising to a power